Analysis and Design of FIR filters using Window Function in Matlab

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ABSTRACT

The input signal has a great influence on the performance of the system in digital control system. Processing of input signal is important to get useful signal. In the processing of digital signal Finite impulse response (FIR) filter plays an important role. Using Matlab the FIR filter is designed and simulated. Different methods like frequency sampling, window function and convex optimization technique are processed using Matlab in the design of FIR filter. By comparing the signal’s amplitude-frequency diagrams which have been generated the filtering effect of different digital filters are analyzed by using FIR digital filters which are designed to process the input signal based on the Matlab function. This paper shows the experimental results show the FIR filters designed.

Keywords: Matlab, frequency sampling, amplitude-frequency, optimization, FIR filter.

1. INTRODUCTION

1.1 Digital filter

It is a discrete system which can do a series of mathematic processing to the input signal, due to which we can obtain the desired information from the input signal. The transfer function for a linear, time-invariant, digital filter is usually expressed as:

\[ G(z) = \frac{\sum_{j=0}^{M} b_j z^{-j}}{1 + \sum_{l=1}^{N} a_l z^{-l}} \]

Where \( a_i \) and \( b_i \) are coefficients of the filter in Z-transform.

There are different kinds of digital filters, and also many different ways to classify them.

Based on its function, the FIR filters can be categorized into four categories, which are low pass filter, high pass filter, band pass filter, and band stop filter.

Based on the impulse response, there are two types of digital filters, finite impulse response (FIR) filters and infinite impulse response (IIR) filters. In the formula above, if \( a_i \) is zero, then it is a FIR filter, else, if there is at least one none-zero \( a_i \), then it is an IIR filter. We need three basic arithmetic units to design a digital filter, which are the adder, the multiplier and the delay.

The following are some steps for designing a digital filter:
1) According to the given requirements make sure of the property of a digital filter.
2) Use a discrete LTI system function to approach to the properties.
3) To design the system function Make use of algorithms.
4) To achieve the task use a computer simulation or hardware.

1.2 FIR filter

Finite impulse response (FIR) filter is the basic elements in a digital signal processing system, and it can assurance a firm linear phase frequency characteristic with any kind of amplitude frequency characteristic. The unit impulse response is finite; so FIR filters are stable system.

1.3 IIR filter

The infinite impulse response (IIR) filter has a feedback loop and is recursive structure, and it. IIR filters are not linear phase and the precision of amplitude frequency characteristic is very high.

1.4 Comparison of FIR and IIR

1. For the same conditions as in the technical indicators, there is a feedback to the output of the
IIR filter from the input, so it meet the requirements better than FIR. It is more economical because the storage units are less than those of IIR, and, the number of calculations is also less in IIR.

2. The phase of the IIR filter is not linear while the FIR filter the phase is strictly linear.

3. In FIR filter finite precision arithmetic error is very small and has a non-recursive structure. While IIR filter parasitic oscillation may occur in the operation of IIR filter and has a recursive structure.

4. Fast Fourier Transformation cannot be used in IIR but can be used in FIR filter.

2. BRIEF INTRODUCTION TO MATLAB

2.1 Matlab

Matlab was released by the The MathWorks companies in the USA, it mostly focus on interactive program designed high-tech computing environment, scientific computing, visualization, matrix calculation, Numerical analysis, nonlinear dynamic system modeling and simulation, scientific data visualization and many other powerful features are integrated in it. Matlab provides a complete solution for engineering design, scientific research, and many other scientific fields. It shows the advanced level in international scientific computing software world.

2.2 Advantages of Matlab

- The program interface is very practical and publishing platform
- The working platform of programming environment is friendly.
- Developing the user interface
- Easy-to-use programming language
- Powerful image processing capabilities
- Collection of modules Toolbox wide range of applications
- Data processing power is very strong

3. Design of FIR filter

It is necessary to specify transition band pass-band, and stop-band, before designing a frequency-selective filter. Frequency need to be pass attenuated in pass-band. In stop-band, frequencies needs to be passed attenuated. Transition band contain frequencies which lying between the pass-band and stop-band. Therefore, the entire frequency range is divided into one or more pass-bands, transition bands, and stop-bands.

The magnitude is not needed to be constant in the pass-band of a filter. Some amount of ripple is usually permitted in the pass-band. Similarly, in stop-band the filter response does not to be zero. A small, nonzero value is also acceptable in the stop-band. The following picture shows some ripples.

![Fig. 1. Low pass filter Amplitude-frequency characteristics.](image.png)

The above figure shows, that the transition band of the filter is between the pass-band and the stop-band. The band-edge frequency \(\omega_s\) defines the edge of the stop-band and the frequency \(\omega_p\) denotes the edge of the pass-band. The ripple is denoted as \(\delta_p\) in the pass-band of the filter, and its magnitude varies from \(1-\delta_p\) to \(1+\delta_p\).

3. WINDOW FUNCTION

In this method, a certain bandwidth is generated using a truncated ideal low-pass filter, and then we use a selected window to get certain stop-band attenuation. The filter length \(L\) can be changed in the transition band to meet a specified roll-off rate. We start with windowed, truncated low-pass filters, then do it for other kind of filters, like band-stop, band-pass, and high-pass filters can also be achieved by several techniques.

Any finite-length of a low-pass impulse response may be considered as the product of a window function \(W\) and the infinite-length low-pass impulse response.

\[
a = \frac{\sin(\omega_c[n-M])}{\pi[n-M]} W_L[n-M].
\]

Here are some of the basic information on standard windows.

i. The simplest window is the rectangular window \(Z[n]\):

\[
Z[n] = \begin{cases} 
1 & 0 \ll n \ll L-1 \\
0 & \text{otherwise}
\end{cases}
\]
ii. The Hanning window;
\[ B[n] = \begin{cases} 0.5 - 0.5 \cos \left( \frac{2\pi n}{L-1} \right) & 0 \ll n \ll L - 1 \\ 0 & \text{otherwise} \end{cases} \]

iii. The Hamming window;
\[ C[n] = \begin{cases} 0.54 - 0.46 \cos \left( \frac{2\pi n}{L-1} \right) & 0 \ll N \ll L - 1 \\ 0 & \text{otherwise} \end{cases} \]

iv. The Blackman window;
\[ D[n] = \begin{cases} 0.42 - 0.5 \cos \left( \frac{2\pi n}{L-1} \right) + 0.08 \cos \left( \frac{4\pi n}{L-1} \right) & 0 \ll n \ll L - 1 \\ 0 & \text{otherwise} \end{cases} \]

v. The Kaiser window;
\[ E[n] = \begin{cases} \left( \frac{a}{\beta} \right)^{\frac{1}{\beta}} \left( 1 - \frac{\pi}{M} \right)^{\frac{1}{\beta}} & 0 \ll n \ll L - 1 \\ 0 & \text{otherwise} \end{cases} \]

5. FIR FILTER DESIGN USING MATLAB

5.1 Realization of window function method by Matlab

firl and kaiserord command is used in the realization of window function in Matlab.
Some definitions of these two functions are as follow.
Function firl: \( a = \text{firl}(n, Wn, 'ftype', \text{window}) \) where \( n \) is the order of filter;
\( Wn \) is cutoff frequency. If \( Wn \) is a two-element vector, \( Wn = [x1 \ x2] \), then it is a band-pass filter with a pass-band from \( x1 \) to \( x2 \). If \( Wn \) is a multi-element vector, \( Wn = [x1 \ x2 \ x3 \ ... \ xn] \), it returns an order of \( n \) multiband filter with bands from \( 0 \) to \( x1 \), \( x1 \) to \( x2 \), \( \ldots \), \( xn \) to \( 1 \);
\( ftype \) shows type of filter, for example, \( ftype='stop' \), it shows a stop-band filter;
\( ftype='high' \), it shows a high-pass filter.
When there is no indication then it shows the default type which is low-pass filter.
window denotes to required window, kaiserord(n, beta) is Kaiser window, hanning(n) is Hanning window, hamming(n) is Hamming window, blackman(n) is Blackman window, boxcar(n) is rectangular window, and when there is no indication by default it is Hamming window.
Function kaiserord:\( [n,Wn,beta,ftype]=\text{kaiserord}(f,a,dev,fs) \)
Where \( f \) is a vector and it is used for the start and ending point of filter’s transition band;
\( a \) is a vector, and is used for the amplitude of selected frequency;
dev is a vector having same length with \( a \), and is used for the maximum possible amplitude error of each pass-band and stop-band;
\( n \) represent the minimum order of filter that can be used to meet the requirements;
\( Wn \) shows the cutoff frequency of filter;
\( ftype \) is the type of filter.

5.2 Design a low-pass filter using the window function method

Requirements:
- Using sampling frequency 2 kHz and a Hamming window.
- Cutoff frequency of stop-band is \( 0.17\pi \) and Cutoff frequency of pass-band is \( 0.1\pi \).
- The stop-band attenuation is \( \geq 50\text{dB} \) and the pass-band attenuation is \( \leq 0.1\text{dB} \).

The cutoff frequency is all normalized frequency, where \( fs \) is the sampling frequency.

![Fig. 5.2a Low pass filter gain response using Hamming window](image1)

![Fig. 5.2b Time domain before filtering](image2)
Comparing Figure 5.2b-2d and 5.2c-e, we see that the input signal is made up of two superposition signals with different frequencies. Stop-band starts from 0.17 kHz and the pass-band is 0-0.1 kHz, and two frequencies of 0.1 kHz and 0.2 kHz required filtered. The signal with 0.1 kHz frequency is not filtered and is kept because it is in the range of pass-band, while the signal with 0.2 kHz frequency is filtered because it is in the range of stop-band.

5.3 Design a band-pass filter using the window function method

Requirements:
- Use sampling frequency is 8 kHz and Kaiser window.
- Stop-band cutoff frequency is 0.6025π and 0.25π and Pass-band cutoff frequency is 0.5525π, and 0.325π.
- The stop-band attenuation is ≥ 40dB, and the stop-band and pass-band ripple is 0.01.
Comparing figure 5.3b-5.3d and 5.3c-5.3e, we see that input signal is made up of four superposition signals and with four different frequencies. The stop-band is from 2.41k to 4 kHz and from 0 to 1 kHz and the pass-band is from 1.3 kHz to 2210Hz. There are four signals with frequencies 0.5k, 1.5k, 2k, and 3kHz and these signal need to be filtered. In figure 5-3e, there are only two signals with frequencies of 1.5k and 2kHz, these frequencies are in the range of pass-band filter so that are not filtered, while the signals with frequencies range from 0.5 k and 3 kHz are in the range of stop-band filter so they are filtered. It shows that the designed filer is working properly.

5.4 Design a multi-pass-band filter using the window function method

Requirements:
- Use sampling frequency is 0.2 kHz and Kaiser window.
- Stop-band cutoff frequency is 0.9π, 0.6π, 0.5π, and 0.1π and Pass-band cutoff frequency is 0.8π, 0.7π, 0.4π, and 0.2π.
- Pass-band and stop-band ripple is 0.01 and the stop-band attenuation is ≥ 30dB.

There are three stop-bands and two pass-bands, the stop-bands are from 0-0.01k, 0.05k-0.06, and 0.09k-0.2 kHz, pass-bands are from 0.02k-0.04 kHz and 0.07k-0.08 kHz, and from figure 5-4c, there are six superposition signals before filtering, and there frequencies are 0.005, 0.02, 0.03, 0.055, 0.075, and 0.095 kHz respectively. Figure 5-4e, shows that signal with frequencies of 0.02, 0.03 and 0.075 kHz are kept because they are in the range of pass-bands, while other signal are filtered out because they are in range of the stop-bands.
6. CONCLUSIONS

Window function is used to design different filters like band-pass, low-pass and multi-pass-band filter. Different windows were selected for filter. When windows were applied on the signal they filtered that frequencies which were not in its range and kept that frequencies which were in it range. So if we want to pass or stop some frequencies window function can used for it. In low pass filter only low frequencies were pass and other were filtered out. Band-pass pass a portion of frequencies and other were filters out. In short window function can be applied to any signal using any type of filter to get desired frequencies.

REFERENCES