Design of a Non Fragile Optimal Control Based Realization in Comparison between LQR and Observer Based Controller for Magnetic Levitation System

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ABSTRACT
This paper explains Magnetic Levitation system of a train which comprises of guidance track made with magnets. The main objective is to compare the output response of the Linear Quadratic Regulators (LQR) and Observer based controller and to observe that which controller can perfectly suspend and propelled the train on a guidance track made with magnets. To perform the desired task state space model of Magnetic Levitation system is derived. The response of the system is simulated in MATLAB/SIMULINK. The open loop response showed that the derived model is unstable. The output response of Linear Quadratic Regulator (LQR) and Observer based Controller are used to analyze the system in closed loop. Both the controllers showed improved performance for different magnetic tracks. It is observed and shown that observer based controller has better response than Linear Quadratic Regulator (LQR) controller. Simulink models for Linear Quadratic Regulator (LQR) and observer based controller are designed. Besides, different types of realization techniques (minimal realization, balanced realization, modal realization, observer canonical realization) are compared for minimum fragility in controller implementation. The difference among the different realization controllers has been analyzed in detail for rounding off error or truncation error and an optimal non fragile controller design has been presented. Different disturbances were imposed upon the simulated model. All the results are analyzed in open and closed loops. The closed loop response showed that the train is suspend and propelled on the track and the desire results were achieved successfully.

Keywords: Optimal Control, Realization, Non-fragile, LQR, Observer Based.

1. INTRODUCTION
Magnetic levitation systems have many varied uses such as in friction less bearing, high speed maglev passenger trains, levitation of wind tunnel model, vibration isolation of sensitive machinery levitation of molten metal in induction furnaces and the levitation of metal slabs during manufacture (Laithwaite 1965, Jayawant and Rea 1965) [1]. Here our objective is to compare the output responses of the Linear Quadratic Regulator (LQR) and Observer Based Controller and to observe which controller can suspend and propelled the Maglev train on the track. The maglev train is based on three type of systems which control the train moves on the track Guidance system, Propulsion system and levitation system [2]. Guidance system refers to the sideward force that requires to move the train on the track. Propulsion system uses an electrically powered motor in the guide way which appears to be the favoured option for high speed Maglev trains. Levitation system keeps the train suspended against the gravity by forces of magnetic field. These systems have unstable open loop response, to make the response of the system stable feedback path was used. Both of the controllers were used to make the closed loop response of the system stable [3].

Valer and Lia build a nonlinear model for magnetic levitation system and proposes systems linearization principle (the expansion in Fourier series and the preservation of the first order terms) in order to linearize the acquired nonlinear model [4-5]. Our interest here is to design a Non-fragile optimal controller so linear controllers were designed to give safety and ride comfort to passengers inside the cabin of a train. The complete mathematical derivation for the Magnetic Levitation train model in state space form is simulated in MATLAB / SIMULINK [6]. The block diagram of the Magnetic Levitation train is shown in the figure 1.
2. MATHEMATICAL MODEL OF MAGNETIC LEVITATION SYSTEM

State space model of Magnetic Levitation is derived as given in [4]

\[
\frac{dv}{dt} = L \frac{du}{dt} + \frac{1}{x} \frac{dx}{dt} \quad \text{(1)}
\]

\[
u = Ri + \frac{dL(x)}{dt} \quad \text{(2)}
\]

\[
m = mg - C \left( \frac{\dot{x}}{x} \right) \quad \text{(3)}
\]

Equation (2) indicates that L(x) is a nonlinear function. Various approximate values are used to determine the value of inductance for the Magnetic Levitation. If we take the assumption that the inductance of the system varies with the inverse of the ball position

\[
L(x) = L + \frac{L_0 x_0}{x} \quad \text{(4)}
\]

Where L is the constant Inductance of the coil in the absence of the ball, L0 is the additional inductance contributed by the presence of the ball

\[
u(t) = iR + \frac{d}{dt} \left( L + \frac{L_0 x_0}{x} \right) i \quad \text{(5)}
\]

\[
u(t) = iR + L \frac{di}{dt} - \frac{L_0 x_0}{x^2} \frac{dx}{dt} \quad \text{(6)}
\]

Substituting \( L_0 x_0 = 2C \) [4], we get

\[
u(t) = iR + L \frac{di}{dt} - \frac{C}{x^2} \frac{dx}{dt} \quad \text{(7)}
\]

Linear Model of the System is

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
\frac{C x_0^2}{m x_{01}^2} & 0 & -2 \frac{C x_0^2}{m x_{01}^4} \\
0 & 2 \frac{C x_0^2}{L x_{01}^3} & -\frac{R}{L}
\end{bmatrix}
B = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
C = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\]

3. METHODOLOGY

The output response of both controllers i.e. LQR (Linear Quadratic Regulator) and Observer Based Controller was compared that which controller response can perfectly overcome the disturbance effect and overcome the disturbing effect and to improve the performance parameter and make the close loop response of the system stable. Different realization techniques are used to obtain a reduced and non-fragile model [3].

3.1 Realization Techniques

In order to obtain a reduced and non-fragile optimal controller different realization techniques are used. Minimal realization (The realization is known as "minimal" as it defines the system with least number of states). Balanced realization, Modal realization and Observer based canonical realization are the other different techniques used to obtained a reduced and non-fragile model.

4. RESULTS AND CONCLUSION

This work was carried out on considering a Magnetic Levitation System. The mathematical derivations were done in state space form. For simulation MATLAB software was used. Several road disturbances were being injected to the system. The open loop response in MATLAB shows oscillations, large overshoot and required large settling time to damp. Different controllers/compensators were designed to obtain the desired response. LQR controller improved the performance of the system. The results obtained were satisfactory. Then observer based controller was designed. After adding the observer gain to observer based controller the performance of the system was improved significantly as compared to LQR. Different realization techniques were then used, by applying these techniques the controllers action was made more efficient and the system was made highly stable and non-fragile.
4.1 Comparison between LQR Controller and Observer Based Controller

![Comparison between LQR and Observer Based controllers](image1)

From the above plot it was very clear that the observer based controller has much better response as compared to the LQR controller. The overshoot and the settling time have been reduced up to a great level.

4.2 Minimal Realization

For LQR controller no state has been reduced, while in Observer Based controller three states have been removed. The controlled response has three states and after minimal realization the states remain the same in LQR controller and for observer based controller three states have been reduced, the controlled response has six states and after minimal realization the states were reduced to three. Difference between the LQR controller, Observer based and minimal realization response was plotted as shown in the figure.

![Minimal Realization](image2)

4.3 Balanced Realization

Difference between the LQR controller, Observer based controller and balanced realization response was plotted as shown in the figure.

![Balance Realization](image3)

4.4 Model realization

Difference between the LQR controller, Observer based controller and model realization response was plotted as shown in Figure

![Model Realization](image4)

4.5 Observer Canonical Realization

Difference between the LQR controller, Observer based controller and Observer Canonical realized response was plotted as shown in Figure

![Observer Canonical Realization](image5)
A brief summary of all types of realization techniques was given below in table 1. This table shows that observer canonical realization on LQR and balanced Realization in Observer based controller gives the least error to controller which represents the most optimal and most non-fragile optimal controller technique.

Table 1: Realization analysis for different controllers

<table>
<thead>
<tr>
<th>Realization type</th>
<th>LQR controller</th>
<th>Observer Based Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal Realization</td>
<td>10-15</td>
<td>10-16</td>
</tr>
<tr>
<td>Balanced Realization</td>
<td>10-15</td>
<td>10-17</td>
</tr>
<tr>
<td>Modal Realization</td>
<td>10-14</td>
<td>10-14</td>
</tr>
<tr>
<td>Observer canonical Realization</td>
<td>10-16</td>
<td>10-16</td>
</tr>
</tbody>
</table>

For different input disturbances the Observer based controller shows better response. The Observer based controller settles the oscillations more quickly, reducing the oscillation and overshoot. The designed Observer based controller provides better handling ability for wide range of disturbances and provides better ride comfort for passengers.

Hence it was very clear from results that the observer based controller shows better response as compared to LQR controller.

Also minimal realization gives the least error to controller which represents the most optimal and most non-fragile optimal controller technique.

REFERENCES